Example 1

cont…

cont…

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cont…

cont…

Examples of birth-and-death process

Example 2:

A machine is operational for a time that is exponentially distributed with rate α and off or down for a time that is exponentially distributed with rate β. For example, the machine needs a part that has an exponentially distributed lifetime; once it burns out, the fix-it time (time required to obtain a new part) is also exponentially distributed.

$$X(t) = \begin{cases} 1, & \text{if the machine is operational at time t} \\ 0, & \text{if the machine is not operational at time t} \end{cases} \tag{1}$$

This is a continuous parameter discrete state Markov process There is no absorbing barrier state.

Instantaneous transition prob.	PMF	Transition Prob.
P_{01}	$P_1(t)$	$P_{00}(t)$ $P_{01}(t)$
P_{01}	$P_0(t)$	$P_{10}(t)$ $P_{11}(t)$

$T_1 \sim exp(\alpha)$ and $T_0 \sim exp(\beta)$.

T_0 - time it takes to repair the component.

cont…

cont.

cont.

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cont.

cont.

cont.

cont.

cont.

Important facts about the exponential distribution (cont.)

Fact 2

If $T_i \sim exp(\lambda_i)$, where $i = 1, 2, ..., n$. Then $M = min(T_1, T_2, ..., T_n)$ has an exponential distribution with parameter $\sum_{i=1}^{n} \lambda_i$. That is, $M \sim exp(\sum_{i=1}^{n} \lambda_i)$.

Fact 3

If $T_i \sim exp(\lambda_i)$, where $i = 1, 2, ..., n$. Then,

$$P[T_j = min(T_1, T_2, ...T_n)] = \frac{\lambda_j}{\sum_{i=1}^{n} \lambda_i}.$$

cont…

cont…

cont…

cont…

Examples of birth-and-death process

Example 3 (One server queue)

Suppose that there is **one** checkout counter at a shopping store. Suppose that customers arrive to a single server system according to a Poisson processing having rate λ and the service time is exponentially distributed with parameter ν.

cont.

cont.

cont.

cont.

Examples of birth-and-death process

Example 4 (Two server queue)

Suppose that there are **two** parallel identical checkout counter at a shopping store. The service times are independently and identically distributed. Suppose that customers arrive according to a Poisson processing having rate λ and the service time is exponentially distributed with parameter ν.

cont.

cont.

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cont.

cont.

Examples of birth-and-death process

Example 5

A telephone operator has a phone with a hold button. Suppose incoming voice calls arrive according to a Poisson process with rate λ. Each call takes an exponentially distributed time with average $1/\nu$ minutes. If a call arrives during a time the phone is busy, it is placed on hold. If another call arrives, it receives a busy tone and must hang up.

cont.

cont.

cont.

cont.

cont.