STA 331 2.0 Stochastic Processes

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Random Walk (drunkard's walk)

We start at 0, then at each timestep, we go up by one with probability p and down by one with probability $q = 1 - p$. When $p = q = \frac{1}{2}$, we are equally likely to go up and down, and we call this the **simple symmetric random walk**.

Simple random walk is very useful to model

- stock prices
- states of populations
- position of gas particles

1. Is this a Markov chain process?

2. Draw the state transition diagram.
3. Write the transition probability matrix.
4. If we start in state 0, what is the probability that after two steps a simple random walk has reached $X_2 = 2$ in two steps?

General Random Walks

An alternative way to write the simple random walk is

$$X_n = X_0 + \sum_{i=1}^{n} Z_i$$

where the starting point is $X_0 = 0$ and the **increments** $Z_1, Z_2 \ldots$ are independent and identically distributed (IID) random variables with $P(Z_i = 1) = p$ and $P(Z_i = -1) = 1 - p = q$

Any stochastic process with the above form for some X_0 and some distribution for the IID Z_i is called a **random walk** (without the word "simple").

Find $E(X_n)$ and $Var(X_n)$

Gambler's rule

Ann is gambling against Benika. Ann is gambling against Benika. Ann starts with USD a and Benika starts with USD b Total amount of money they both have is $m = a + b$.
At each step of the game, both players bet USD 1; Ann wins USD 1 off Benika with probability p, or Benika wins USD 1 off Ann with probability
q. The game continues until one player is out of money (or is "ruined").

Let X_n be how much money Ann has after n steps of the game.

1. What is the state space and parameter space?
2. Is this a Markov chain process?

3. What is the probability that the game ends by Ann ruining?
4. How does the game last on average?

Take home task

1. Linear difference equations
2. Homogeneous linear difference equations
3. Inhomogeneous linear difference equations