

STA 331 2.0 Stochastic Processes

Important results: Exponential distribution

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Fact 1

If $T \sim \text{exp}(\lambda)$, then $P(T > t + s | T > t) = P(T > s)$. This is called the **memoryless property**, also known as **lack of aging property** or **lack of memory**.

Exponential distribution is the only continuous distribution with the memoryless property.

Geometric distribution is the only discrete distribution with the memoryless property.

Proof: In-class

Fact 2

If $T_i \sim \exp(\lambda_i)$, where $i = 1, 2, \dots, n$. Then $M = \min(T_1, T_2, \dots, T_n)$ has an exponential distribution with parameter $\sum_{i=1}^n \lambda_i$. That is, $M \sim \exp(\sum_{i=1}^n \lambda_i)$.

Proof: In-class

Fact 3

If $T_i \sim \text{exp}(\lambda_i)$, where $i = 1, 2, \dots, n$. Then,

$$P[T_j = \min(T_1, T_2, \dots, T_n)] = \frac{\lambda_j}{\sum_{i=1}^n \lambda_i}.$$

Proof: In-class —

Fact 4

Suppose that $T_i \sim \exp(\lambda_i)$. Let $k > 0$ and $T_2 = T_1 - k$. If $T_1 \geq k$, then $P(T_1 > t) = P(T_2 > t)$.

Proof: In-class